

Running head: Critical Thinking

Stimulating Critical Thinking Through Mathematics Word Problems

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STIMULATING CRITICAL THINKING THROUGH MATHEMATICS WORD  
PROBLEMS

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### Abstract

Mathematics is one of the most important subjects taught in all schools, it has a universal language that requires reading comprehension skills to be able to solve word problems. This action research reveals the use of critical thinking strategies with second grade students attending academic learning support, in a well recognize IB bilingual school in Bogotá. The purpose of this research is to support students attending mathematics learning to develop mathematics thinking skill through the comprehension of word problems in a second language that is English. With the implementation of thinking strategies such as to reading the problems and look for meaningful information, underlining key words and by visualizing the problem, students stimulate their learning process and become critical thinkers in the comprehension of mathematics word problems. These strategies tended to increased students confidence in solving word problems and become more competent mathematicians.

*Key words:* Critical thinking, Comprehension, Visualization, Content-based, Learning.

## Introduction

The English School (TES) is a private co-educational institution of bilingual, international education, whose aims are to develop the highest possible standard, by emphasizing the full development of each student's abilities. Therefore, the school is committed to help any students who need Learning Support of any kind to overcome their difficulties and to achieve their full potential. In particular to those students that need special reinforcement in any area an academic support, to overcome their difficulties and to achieve their full potential.

As a teacher in this school, I have realized that many students that attend mathematics learning support are students that struggle over understanding word problems in mathematics. For that reason, this project focused on building conceptual supports that include strategies to be carried out such as recognition of the function of key words, word phrases in a word problem and also the importance of making drawings to visualize concepts to help them comprehend the mathematical situation. Critical thinking skills such as gathering, organizing, remembering, and analyzing information in a word problem are intended to change students' perception towards the mathematics and improve their academic standards.

This project was developed within the line of inquiry "Desafíos contemporáneos de la enseñanza y aprendizaje de lenguas extranjeras" (Contemporary challenges in teaching and learning foreign languages) that offers opportunities in a society which demands new competencies for an optimum education.

Students attending learning support program have very few skills when dealing with understanding word problems. They seem to have difficulty to comprehend the “situation” which forms the context of the problem. My challenge here is to teach how to recognize word problems situation and at the same time to implement strategies to connect word problems into a real life situation, which will encourages students to analyze their own comprehension process and eventually to come out with an answer. Reflecting upon their own thinking process will help those not only in the mathematical field but also these abilities can be transfer to any subject.

Therefore for students to become critical thinkers in the comprehension of mathematics word problems, they are required to develop understanding about the words in the problem which will benefit by functioning productively in their classes and help them to be critical problem solvers.

### Statement of the Problem

For many years as a teacher in “The English School”, a bilingual school in Bogotá, I have noticed the difficulty that some students have in understanding and analyzing mathematical word problems and at the same time in connecting aspects of mathematics with their real life. As a result, these students are sent to the Learning Support Department to receive additional help to meet the essential standards required for the level, due to their difficulties in solving word problems, which requires from them a clear understanding of how to deduce the information from a written text given in a word problem and to be able to use this information as a way to arrive at a solution.

This project offers some specific suggestions for making the mathematics learning process in the academic learning support a place where students are able to improve their thinking skills by guiding them to perform at their best, and be able to reach the school academic standards. There is a need to rebuild the students’ self confidence and for this reason it is fundamental that students attending learning support find motivating strategies to target their problems.

Furthermore when rebuilding the students’ self confidence by encouraging them to get into the habit of analyzing when reading a word problem, increased students’ self-confidence , I have also noticed that these students’ mistakes come mainly from misinterpreting instructions of the questions and in this sense, there is a need to have students restate the problem and have them describe how they arrived at the possible solution and how they are grasping the problem and relating it to the real world.



Critical-thinking strategies are ways that lead students to connect and understand what they are learning, it is an important role to develop in the students' abilities to understand and solve word problems. Students should be taught methods to reason and think critically about a problem, through the identification of key words and phrases and by drawing visual aids that enable them to decide what to do. When reading the question and simplify the word problem by highlighting the main words or an important idea, or drawing the situation of the problem, they are generating critical thinking skills that can be used in any other academic subject taught in a school.

In order to improve students' critical thinking skill, teachers need to find ways to integrate opportunities for increasing critical thinking into regular lessons. Regarding to mathematics as Polya (1973) mentioned that in order to understand a problem, the students should devise a plan, carry out the plan and finally look back. It is important that students acquire mathematical understanding should construct their own meaning through exploring their own personal experience, and by using it in the real life situation; mathematics needs to be taught in relevant, realistic context. As Polya argued mathematics problem should never end just because the answer has been found, but it is essential to go beyond that as a form to expand students' critical thinking skills. There should be a constant questioning related to the problem.

## Research Questions and Objectives

### Research Question

As a result of the observation of students' attending Academic Learning Support, the following research question and objectives were used in developing my action research project:

#### Main question

How does the implementation of visual representations and the recognition of key words and phrases in mathematics word problems in an L2 contribute to the development of critical thinking skills?

#### Research aim

To describe how the implementation of visual representations and the recognition of key words and phrases contribute to the development of critical thinking skills.

#### Specific research objectives

- To implement visual representations strategies to improve students' critical thinking skills
- To help students recognize key words and phrases to achieve a better understanding of word problems

### Justification

The competence to think critically has nowadays become a challenge. There is a need to reflect upon actions, encouraging young learners to use the ability to face daily word problems and to prepare them for real life which requires logical and abstract thinking skills. The purpose of my study is to accompany and develop thinking skills in students at The English School attending the academic learning support department, where it is clear that students find it extremely difficult to understand and analyze written word problems given in the regular class, because they have not developed the skills to analyze, deduce and infer conclusions from a word problem.

Even though The English School is a bilingual school with high academic standards, this does not mean that all students are able to think critically. Therefore, to help these students from second grade attending learning support to be more critical, activities will be designed that require higher level analysis and thus helping them to improve their thinking skills. These include recognition of mathematical concepts by developing different strategies for solving contextual word problems and to improve their performance, understanding the comprehension of word problems, by posing solutions where the aim is not just to focus on the surface aspect of the problem or the numerical form but instead working deeply into its essence of the question formulated.

Bearing in mind that the primary years are extremely important in children's intellectual development, there is a need to encourage students to think critically by showing them alternative ways to solve a problem, in order to progress in their cognitive skills. At the same time the aim is to promote more proactive learning where they feel motivated by finding different ways to solve problems in mathematics. This will guarantee

them a more successful academic performance where they will be able to face up to new challenges.

Consequently this research project is focused on helping these young learners from this international school to improve their thinking skills and prepare them to cope with the school academic demands. With the results of the present research, The English School's an principally Learning Support Department will benefit from this project in order to incorporate critical thinking strategies as something to be taught deeply to improve students thinking skills of the primary pupils who are struggling to cope with the demands and not only focused on the academic demand of the school curriculum.

Becoming effective thinkers is important especially when students are expected to cope with a bilingual education and be prepared to function in a multicultural world. Reasoning critically gives students the ability to not only understand what they have read on a word problem of mathematics, but also to build upon that knowledge and to take action in their real life situation.

This can only be achieved through the process of analysis and understanding which requires students to go beyond the information given in a text, by connecting key words, ideas and concepts, so they can devise their own solution to a problem and learn to think critically. Teachers can aid students in this goal and build students knowledge so they will become individuals who are able to think by themselves and not just blindly apply mathematical formulae they have memorized but do not understand.

As Stewart (1996) has written Mathematics is about ideas. In particular it is about the way different ideas relate to each other. "The aim of mathematics is to understand the questions by stripping away the inessential and penetrating to the core of the problem. It is not just getting the right answer; it is more a matter of understanding why an answer is

possible at all, and why it takes the form that it does” ( p. 20 ).this is a very important view to have in mind in this project, which confirmed that mathematics is much more than doing an operation, it is about constructing meaning , transferring meaning and applying with understanding. Therefore, teaching students to become skilled thinkers is a goal of education and one aim of education should be focused on students’ capability to solving and understanding mathematical concepts so they can develop higher cognitive skills. This requires appropriate challenges to extend their learning which take into consideration student’s zone of proximal development as mentioned by Stewart(1996) where he stresses the need to consider not only the child’s current level of cognitive functioning but also the child’s level of “potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p.86). Therefore we need to accept that students’ low performance in mathematics should be attributed more to our failure to motivate students to succeed in their mathematics performance rather than their lack of innate mathematical ability.

### Literature Review

The main objective of this project is to guide students into developing their critical thinking skills through the use of mathematics word problems, as it is defined in this action research project. The idea is to reinforce the learning of young students from second grade level who are attending Academic Learning Support using a wide range of word problems to develop their thinking skills. Although some students may be more advanced than others, the point of this project is that everyone without exception can acquire a reasonable amount of critical thinking skill if there is the desire and the willingness to do so. Bono (1991) explains that thinking can be improved through attention and practice, until the skill is mastered. For that reason, it is my goal in this action research to help students to solve word problems successfully by working with specific strategies as visualization of the problem and to understand word in a problem. Therefore I took into consideration theories regarding learning mathematics in a second language in order to give a foundation to this project. The consideration of different theory helps to provide support to guide this research regarding language acquisition and critical thinking which have an important role in the formation of successful students. The action research follows the concepts of constructivism, content based language learning and critical thinking as a way to achieve the objective of aiding students to understand mathematics word problems.

Content based instruction is focused on the teaching and learning in a second language rather than learning about language directly. Therefore, the goal is to integrate mathematics content with English language instruction and to promote critical thinking

skills. It is important to note that in content-based instruction, students acquire the content area of the subject matter with understandable input, and at the same time increase their language skills. To achieve the goal of language skills improvement, Krashen (1982) remarks that teaching must be meaningful and authentic. This project is taking his idea and applying it to the acquisition of a second language. The main objective is to create strategies to support the students' successful acquisition of the L2 by way of a creative and meaningful learning route. Mathematics word problems can be confusing for students who have difficulty conceptualizing the purpose of the presented situation. However, when students are able to understand and solve word problems, they can begin to understand how to use their math skills in their daily lives.

Mathematics word problems are filled with key words and clues that will help you understand the underlying equation. Identify words that provide the use of the numerical identification like: added, more than, equal to by underlining throughout the entire problem is an strategy that offers students comprehension on the problem, and by visualization are strategies that develop that increase students critical thinking and enable them to learn in a second language, students follow content processes on sequential levels such as, visualization, analysis, deduction and rigor. As stated by Van Hiele(1986), creating an understanding of the language in content instruction allows opportunities for learners to acquire a new language through the study of academic disciplines such as mathematics and that also includes activities where students are required to think critically and to be able to build problem solving skills. Furthermore, understanding mathematics as a language requires it to be connected to mathematical concepts; that is to say, ideas which explain the "how" and "why" of mathematics. In other words, mathematics word problems often

involve both concepts and skills. Students need to understand concepts to decide how to go about solving a problem. While the concept involves understanding; for example *key words* and *word phrases*, skills involve doing mathematics, *recalling basic facts* and *following procedures* to improve the practice of solving word problems.(Pearson Education Inc. 2006)

Students learn mathematics by constructing their own understanding. They learn mathematics by an active participation, and this can be assured by applying strategies and by creating students interest, for this purpose teachers need to know students and students' prior knowledge make them participated in their own learning process. Constructivism is undoubtedly a major theoretical influence in science and mathematics education. Students need to construct their own understanding of each concept in order to develop their own thinking skill so that the primary role of teaching is not to transfer knowledge, but to create opportunities for students to construct their particular mathematical concept as explained in Piaget's (1951) theory of knowledge. The constructivist Vygotsky's (1978) theory emphasized the aspect of the individual development to construct meaning within a context. Therefore, learning focuses on how student's learning is affected by context, beliefs and attitudes. Here is where the constructivist approach takes place to teach for understanding. Mathematics word problems is a way to help students understanding due to the fact that it provides meaningful learning for students, by making knowledge more familiar and easier to appropriate in a real-life context. According to this research, teachers must use a wide variety of resources among which are manipulative materials such as unit fit, pictures, charts, etc. All these materials not only provide the student with interesting and fun activities to work on, but also engage them in the process of becoming critical thinkers. As



Kamarul (1999) states, producing critical learners is not an easy task, but it can be achieved by engaging the “Pedagogy of Question”, which was proposed by Freire (1970 & 1973), where the methodology requires questions to be posed to learners and listening to learners' questions and answers. This challenges students to think critically and to adopt a critical attitude towards their own learning process.

Another aspect is mentioned by Cobb (1988) where he suggested two major goals for mathematics instruction. The first one is based on students' development in mathematical structures such as meaningful problems. The second requires students to become more in control of their mathematical activity as a way to reinforce mathematics thinking by the use of explorations and participation. Constructivism challenges the basic understandings of content knowledge and reinforces the idea of teaching mathematical thinking through problem-solving contexts. This is well analyzed by Lester (Lester et al., 1994, p.154) where he stated that “helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying aids the improvement of thinking skills and has acquired a high priority in mathematics instructions”.

In this project, the intention is to offer some specific suggestions to make mathematics word problems a way to involve students to learn about the language that carries out, and for them to see it as a more attainable and at the same time to improve their thinking skills as a way to become better problem solvers.

The concept of teaching mathematics skills through word problems solving allows students to be focused more on the concept being asked rather than memorizing facts.

When working on word problems, the goal for both teachers and students is to select as correctly as possible the operation that will provide the answer (Graeber and Tanenhaus 1993). Through experiences of critical thinking, students select the information required and analyze it in order to come to a conclusion. This is a way to develop their critical thinking techniques to evaluate information and ideas, and as a way to decide what to accept and believe (Kurland, 2000). The concept to bear in mind about critical thinking is, “Critical thinking has been defined as learning to evaluate, draw inferences and arrive at conclusions based on the evidence”(Nemat, 2008). The process of critical thinking begins when students can analyze ideas, make inferences, evaluate and conclude. Critical thinking skills involve reflection on all the aspects of the situation in a problem. It includes gathering, organizing, remembering, and analyzing information. This includes the ability to read with understanding, to identify the key words and to draw proper conclusions (Krulik and Rudnick (1993). One way to expand students’ critical thinking skills is by using a wide variety of word problems situations and by extending the problem beyond the answer, for students to reflect and generate new end products (Krulik and Rudnick, 1993).

As stated in the research developed by Nemat (2006) Critical thinking involves the application, analysis, synthesis and evaluation of information so it can be used to construct personal meaning. Critical thinking is described as the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing and/or evaluating information gathered from, or generated by, observation, experience, reflection and reasoning.

Just as it was described in the project, critical thinkers should be taught to think logically, analyze and compare, question and evaluate. In activities that emphasize

thinking, the aims must include application, analysis, divergent thinking and opportunities to organize ideas and support value judgments. McMillen (1986).

There are two ways to teach critical thinking. The first is for the teacher to change paradigms and modify the teaching and testing methodology with the purpose of creating innovative activities that will strengthen students' abilities in critical thinking. The second refers to all the material that can be of support for the teaching of these critical skills. Therefore, as mentioned by Alexander, R (2004), it is essential to have in mind the following features as a way of teaching the development of critical thinking skills in mathematics:

Asking questions:

- Prompts that facilitates mathematical talk
- Opens up new areas for students to think about
- Encourages students to reason about their mathematics
- Requires students to communicate their thinking to others
- Generates a rough indication of current thinking

Demonstrating or modeling mathematical ideas:

- Provides an insight into how to carry out mathematical procedures
- Provides a visual explanation of a mathematical idea
- Models ways of acting mathematically

Evaluating and correcting students' responses:

- Helps children to understand mathematically
- Models mathematical expectations
- Values students' ideas

Since mathematics is fundamentally about solving problems it is basic that students figure out a strategy that will effectively solve the problem. Sometimes students need to construct the strategy for themselves and at other times, they need to choose from the available strategies that have previously been given in class. Solving a word problem requires a flexible process. Students need to set up strategies to build a mental representation upon which the strategy will act. In order to describe the students' problem solving behaviors the following strategies from Artzt and Armour Thomas (1992) were used - to read, understand, explore, analyze, plan, implement, and verify. In order for the teacher to detect instances of reasoning, these are defined by Thornquist (1993) as, "Mathematical reasoning is part of mathematical thinking that involves forming generalizations and drawing valid conclusions about ideas and how they are related" (p.43)

Mathematical reasoning requires analytical thinking, and also requires creative and practical thinking performance. Research has demonstrated that students are capable of powerful mathematical reasoning Grouw (1992) however conventional instruction, rather than building on student's thinking ability about mathematics, centers instead on the memorization of facts. Cohen, McLaughlin, and Talbert (1993) believe that as a result, of this process of schooling, students tend to lose motivation.

There has been research carried out where it clearly indicates that critical thinking is neither inborn nor naturally acquired. Critical thinking must be taught and learned in schools, that is why there is a need to develop in students from a young age their understanding and identify the specific critical thinking skills involved. The research paper “Critical Thinking: Why is it So Hard to Teach?” by Willingham (2007), provides ideas to support my research which aims to develop students’ critical thinking in the solution of word problems by helping them to adjust their strategies on how to understand them by questioning what is the situation and searching for important words and putting the problem into a real life context.

I think that this pedagogy will help students to develop these critical thinking skills but as stated by Kamarul (1999), the current situation is that teachers widely practice the pedagogy of answers, whereby teachers provide the answers and solutions to learners. Most frequently, this is done unconsciously. They do not realize that they are "spoon-feeding" the learners most of the time. By giving answers, teachers deny the learners the opportunities and the right to question, to doubt and to reject. In addition, the learners will not be exposed to challenges and stimulation of thoughts. Friere (1972) added that teachers tend to adopt the pedagogy of answers because they are sometimes afraid of questions to which they are unsure of the answers. Thus, it is vital that teachers adopt new strategies in order to enhance students learning by being more like a facilitator in the learning process. In this sense, critical thinking authors would argue that by helping to make people more critical in thought and action, critical education can “increase freedom and enlarge the scope of human possibilities.” Burbules & Bork (1999).

All these theoretical studies help to see how important it is to develop critical thinking in young learners, and that critical thinking is not innate but that it needs to be acquired. Therefore as I have repeatedly emphasized, critical thinking has to be taught and learned. The aim for teachers is to help students to reflect upon their own thinking processes, and in doing so, to become more successful, active learners. Furthermore, Vygotsky (1978) has stressed the need to consider not only the child's current level of cognitive functioning but also the child's level of "potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p.86). Vygotsky believed that it is important to examine the child's "dynamic mental state, allowing not only for what has been achieved developmentally but also for what is in the course of maturing" (p.87). Here is whereas teachers we need to understand the importance of dealing with mathematics word problems and modes of learning as in part an expression of the individual personality. An understanding of this "personality of cognition" can motivate the child, and enables the teacher to teach the child more effectively. Yet educators often fail to recognize, nourish, and promote mathematical abilities, particularly to those who are disadvantaged.

How can we move the students being into being active participants in the change? Well, by ways to integrate opportunities for increasing critical thinking into regular mathematics lessons. One specific suggestion is by involving them in daily mathematics word problems, emphasizing key words, and word phrases, also the construction of visualizations to allow students to see the problem in a real life context, to improve their creativity and at the same time give them opportunities to enjoy solving problems. Effective mathematics teaching requires a serious commitment to the development of students'

understanding of mathematics. Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know so they can build on that knowledge. Throughout this process students seek to validate their own thinking skills. In addition to curricular demands Behrman (2006) notes that in all cases, the dynamics between students and teachers are also important. Similar to Freire's notion that teachers should be learners and learners should be teachers, a classroom that acknowledges the theory of critical thinking must also challenge traditional hierarchical relationships between the students and teacher.

As Kamarul (1999) refers to in his research, producing critical learners is by no means an easy task, but it can be achieved by engaging the "Pedagogy of Question", which was proposed by Freire (1970 & 1973). This pedagogy requires posing questions to learners and listening to learners' responses. This is a practice which forces and challenges the learners to think creatively and to adopt a critical attitude towards the world.

I think that this pedagogy will help to create critical thinking skills as well, but teachers need to believe that their major roles are to guide, initiate, facilitate and encourage the learners. This will put them in the right frame of mind and lead the institution into becoming a community of collaborative inquirers.

Critical thinking such as Swartz (1992) would argue that by helping to make students more critical in thought and action, this helps our students to develop their critical thinking skills, essentially when dealing with word problems in mathematics, also he emphasizes that students should not only employ critical thinking in a subject but to reinforce this skill in their real-life situation.

Creative thinking is a personal construction of meaning, therefore all the disciplines must aim to develop cognitive flexibility, as Richart(2002) argues that critical thinkers use logic to solve problems. The strategies deal with the issue of clarifying the problem and its components, by considering a viability of alternative for the solutions, and by having a set of steps to achieve those solutions. Where it is evident in the investigations related to helping at risk students solve mathematical word problems through the use of direct instruction, which examines the positive influence of clear strategies to follow as a way to facilitate the understanding of problems and the comprehension of the problem.

*“As students attempt to solve rich problem tasks, they come to understand the mathematical concepts and methods, become more adept at mathematical problem solving, and develop mathematical habits of mind that are useful ways to think about any mathematical situation.”*

Teaching Mathematics Through Problems Solving

NCTM, 2003



### Research Design

From my experience as a teacher, I have always been concerned about the difficulty primary students have when dealing with mathematical word problems; especially those students attending academic learning support program. Consequently, it is here where my research main question.

#### Main Question

How do the implementation of visual representations and the recognition of word phrases and key words in mathematics word problems in an L2 contribute to the development of critical thinking skills?

#### Type of study- Qualitative Action Research

Since the questions of the project arose from real practice it was appropriate to use action research as the tool that teachers might use to solve these problems. “When teachers reflect critically on the teaching situation, identify learning or instructional problems, and institute methods to solve them, the classroom action research takes place”. (Elyildirim & Ashton,(2006). This means that the step-by step process is constantly observed over periods of time and by a range of instruments to be adapted or modify so it brings advantage to the process carried out. (Cohen and Manion, 1980).

Judith Bell (1987) says that action research is an approach which has proved to be particularly attractive to educators because of its practical problem-solving emphasis. It is useful because it deals with the understanding and improvement of practice over a period of

time. Therefore, this action research project is carried out within the context of a school environment on questions that deal with learning difficulties that affect students directly. Bernardo (1997) recommends the use of variable problem contexts to promote abstraction. He states about presenting concepts in variable problem contexts, students will come to appreciate the meaning and use of a particular concept or procedure in a variety of contexts” (p.12). Word problems understanding cannot be recognizing in a single problem, there is a need for wide range of varied problems to facilitate the abstraction of specific concepts and transfer of knowledge to the contexts. He proposed that teaching strategies that promote analogical transfer.

This study is a qualitative approach because it attempts to analyze the effects and consequences that follow pursuing answers to problematic situations. Besides, the process adopts a qualitative perspective mainly because it is more concerned about understanding individuals’ perception, and seeking for insight rather than statistical analysis. Bell (1987).

The essential steps include exploring and identifying a problem in the classroom by conducting a piloting prior to the project as a way to observe students’ perceptions towards mathematics word problems, there were taken randomly 25 students from The English School, from second grade, 10 girls and 15 boys, and given word problems as a story related to tadpoles, it took two sessions, where it was observed that most of the student had difficulty in understanding word problem, and most of them did not follow any kind of strategy. (Appendix A). Afterward there was a survey to perceive students’ feeling toward the problems on which 16 mentioned that they do not like to do mathematics word problems. Based on the result from the piloting, I was more interested to carry out this project and focused the research on how to make students understand word problems in an

pleasant and relaxing way, so they feel that they gain the support that they were needing. It was obvious the necessity to reach comprehension through word problems and to stimulate their critical thinking skill. I started this process as soon as possible with observations done at the beginning of September and planned to apply the strategies little by little after motivating them to deal in a different way toward the solution of mathematic word problems, and by promoting a constant reflection on the word they encounter, all of this warm up motivation was the starting point for the proposal on how to implement strategies that will help students attending learning support to overcome difficulties when working with word problems.

### Setting

The setting of this research project is The English School, a calendar B school where students begin classes in August and finish their school year in June. It is a co-ed school located in the North of Bogotá. The English School (TES) is a private foundation that provides a broad based and rigorous bilingual education in a coeducational, non-denominational and international learning environment. In order to maintain the highest possible international standards, the school offers all the programmes of the International Baccalaureate (IBO) suite: The Primary Years Programme (PYP), The Middle Years Programme (MYP) and The Diploma Programme (DP).

The school's mission is to satisfy the need for a bilingual education to Colombian and foreign students. The English School promotes teaching through the student's personal experience, fomenting integral education through student investigations, with a balanced study plan where the students training involves emotional and cognitive aspects, as well as the development of a healthy body.

The school's vision is to be recognized as a standard-bearer for socially and environmentally responsible leaders in Colombia and the world. Therefore the school's goal is to provide comprehensive education based on quality and excellence, certified by international standards. To develop creativity and innovation, encourage critical and reflective thinking in the students for the practical application of academic contents in community service as well as to develop skills in communication and information technology. For this study and to comply with the ethical demands of the project, permission from the school's headmaster was requested (See Appendix B). After the authorization was given, parents received a letter in which they were informed about the purpose of this research, that is to improve students critical thinking skills when working with mathematics word problems, it seem very appealing to the parents that all of them returned the necessary consent. (See Appendix C).

### Participants

Students (boys and girls) from 2nd grade were the active participants in this project. They are between 7 and 8 years old. These pupils belong to a high socio-economic segment of Colombian society and most have had the opportunity to travel outside of Colombia. The parents are almost all educated to university level. The group is a mixed ability one, which means that the teacher needs to handle different levels of English (low, standard and high). There are total of 10 students in this group made up of three girls and seven boys.

### Instruments

The instruments used to collect data for this study were students' products artifacts, pre and post questionnaires, and personal interviews.

### Survey (See Appendix D)

These three surveys were used during the process at the beginning, middle and at the end of the process as a way to predict, compare and contrast outcomes. Through it I could identify, in the first instance, the students feeling toward mathematics and their interest about this subject and verify the outcomes at the end of the process. The first survey was designed in English because this is the language they are in contact with. Besides their feelings towards mathematics, it was related to their handling of word problems, recognition of key words and phrases. The survey was structured with four yes / no question, an open question and a multiple choice section, each one of them with the objective to know about their related mathematics related weaknesses and also the perception they have about solving word problems. The second survey was also written in English and was designed to contrast their feelings from the starting point up to the middle of the intervention. (See appendix E). The last survey was related to finding out the outcomes about the process, their feelings toward math problems. There were six yes or no questions which were used to appraise their awareness through this experience.

### Students' artifacts

Students provide themselves with a process of learning how to solve word problems by visualizing sequences in which students make drawings to give reality to the situation. The elaboration of the drawings is called "making the movie" and they are a visual representation of their progress in solving the problem. These students' artifacts are all students productions when solving problems that gave me information about the process they used to work out the word problems, all this artifacts were collected through the whole

process, as evidence of their work. One of the aims of this study is to analyze the role of self-explanation in solving word problems. How students read the problems and the way they identify key words and phrases necessary to understand the problem also to see the use of different strategies carried out to solve a problem. Students should be encouraged to think of word problems as real situations. When the students have made sense of the problem, they plan how to solve it, and guiding students to reflect on different images will be particularly constructive for them. (Hyde, 2007).

### Interview

According to Kvale (1996), the main goal in feedback forms is to collect information in order to understand what the interviewees are thinking and feeling. Mcnamara (1999), states that specific questions in interviews are particularly useful for obtaining the information behind the participant's experiences.

The final interview for this research project was carried out as an important complement to the rest of data collection instruments used in this research project. It was designed according to Kvale (1996), as a task to receive to collect information which is used to understand what the interviewees say and to avoid confusions. The questionnaire had five questions that attempted to know the students' opinions about the word problem activities in which the implementation of the different strategies to solve word problems were used. At the same time, this final interview let me identify relevant aspects such as motivation, learning process improvements and general perception of the students in learning critical thinking through word problems. It is a useful instrument because students prefer sharing their ideas orally rather than in written form and so the teacher will understand more fully

the pupils' abilities in thinking processes. As Katherine Keber (1994) reported, transcribing the interaction enables us to arrive at a detailed analysis of the thinking skills shown by the learners as well as the interactions and sharing of ideas.(See Appendixes G

#### Time Line

- First session (September 2010). The first survey was conducted as a starting point for this research project.
- Second Session (October 2010). Applying the second survey
- Third session (November 2010). Students artifacts. Applying the last survey
- Fourth session (January 2011) Students artifacts
- Fifth session: (February 2011) Applying the last survey
- Sixth session: (March 2011) Interviews

### Instructional Design

The present pedagogical intervention is planned with the objective of giving an insight to the students' performance in terms of their critical thinking skills when dealing with mathematical word problems language. For that reason, I have selected a variety of written word problems for students to work on. Each word problem reinforces a critical thinking ability, such as reading and understanding the main question or identifying key words and phrases.

Explanation of the research problem, objectives and methodology, were the research basics required to create an instructional design in which the intention is to investigate several questions during the time students are involved with the mathematics support department.

#### Main Objective

This project was aimed to provide students with critical thinking skills through the use of mathematics word problems analysis, for students to become aware of the purpose of "key words and word phrases" where the students' role is to understand word problems by taking a critical opinion.

#### Specific Objectives

1. To develop critical reading skills such as inferring, predicting, interpreting and evaluating.
2. To develop reflection on the analysis of different kind of word problems.



## Methodology

The pedagogical intervention has been carried out taking into consideration a Content Based approach in mathematics as a way to help students understanding of the L2 language, students benefit from instruction that helps them look critically at texts.

“Understanding a paragraph is like solving a problem in mathematics. It consists of selecting the right relations, and also with the right amount of weight or influence or force of each” (Pearson, 2003, p14). Constructivism was an essential part of this project, where students construct their own work, by taking pieces by pieces of the word, at first it was like a puzzle until they had their hand on to construct the meaning of the problem. Once students comprehend the word problem and construct their own work they will be able to reason critically. The study took six sessions of an hour. The first intervention took two sessions an hour each, where it was focused on the reading aspect of the word problem on their understanding and can identifying common patterns in the problem, word meaning and comprehension of sentences, my interest was based on the thoughtful understanding of the problem and not to reach to a solution. Second sections took three sessions of an hour each where the focus was to visualize the problem, students created different artifacts where they placed on the paper the situation of the problem, their perception of the problem with pictures and like they said by making the “movie” was an strategy to connected the problem with real life, the following session was an hour where students were free to solve the word problems using what ever strategy will help them solve the problem, this was done as a way to confirm the strategies used by them and the acquire of the skill to solve mathematics problems.

A relevant aspect of this project is to consider student motivation to succeed in the different tasks. It is essential to count on a positive attitude on the part of the class.

“Success and satisfaction are key factors in sustaining motivation. If students feel they have achieved something worthwhile, through their own individual effort, they are more likely to participate the next time. Hence the need for teachers to set achievable goals, and to highlight students’ successes”. (Willis, 1996, p14).

#### Strategies and techniques

The activities of this project were carried out during sequences of teaching practice. It was essential to help the learner with the development of critical thinking skill through the use of word problems. The activities were developed by the learning support assistants. The starting point was to focus on the reading comprehension of a word problem, and discuss what they already knew. As Durnin (1997) says, students who receive instruction in word problems solving processes show better performance on tests skills, tasks, and problem solving, as well as on measure of the transfer of learning.

Therefore it is important to mention here by using the right strategy for a word problem solution is at least as important as getting the right answer, that is why it is important to ask “Wh” questions such as what they understand about the word problem; what is the problem asking and what situation needs to be solved. All of these activities aim to increase students’ comprehension and connect to the new information. It was asking questions were the aim was to encourage students to read and think carefully; by reading and thinking about the word problem students make inferences as they read, cross out unnecessary information; look for key words or phrases that give them clues as to how to understand the

problem. These efforts increase inquisitive and analytical thinking right from an early age which are very crucial for solving mathematics problems. In many problems there are certain important parts, some redundant information and a particular knowledge set required. The next part is determining exactly what is required to be done or what is asked in the question. Following step is to read the entire question again and analyze all the relevant information that can assist in the problem. Once they have understood and lay down all information, solve the problem carefully and verify whether the answer obtained can be a logical solution to the question. It may be possible for some students not to get the answer in the first, second or even the third attempt. But constantly facing the task and challenging it is the key to learn how to solve math problems.

#### Key words and word phrase

Identifying of key words and word phrase also helps students to develop their ability to reason critically and translate those key and phrases into mathematical equations.

MacGregor (1993) determines, that "Mathematics teachers are beginning to realize that every math's lesson should be also a language lesson, providing opportunities for all students to develop their English language skills" (p. 31).

Therefore, by students connecting word and word phrases from the problem to a real life situation, and then by the strategy of visualization, students' minds is what makes the problem real for them.

Math word problems can contain a great deal of information in a short paragraph, so finding the right information and making the correct choice takes some skill. Breaking

down the process of solving math word problems will help to get through the paragraph and arrive at the correct answer.

#### Visual aid

As visualization it is a strategy gives the chance to students to think back and recreate the problem. It helped students put the word problem into a real life situation. Math word problems can be a real, especially for students who struggle with reading comprehension or for those learning a second language. Once students develop the habit of visualizing the problems these tasks will stimulate their math problem solving skill and promote critical thinking as a way to go beyond the superficial of the problem and discover that math can be interesting and enjoyable at the same time.

In addition, it is important to take into account that students need guidelines on how to think critically, in order to structure what they already know, perceive or feel. These structures, according to (Short 1993, p 24), “can make sense of the world”. A subject like mathematics can be considered an instrument that offers students an alternative way to examine their surrounding without focusing just on isolated concepts and facts.

According to Vaidya(1970), the goals of mathematics’ education are effectively achieved if students base their work on their personal questions and the problematic situations encountered by them in their own immediate environment. It is important to create an environment in which students can see the language as something natural and common.

## Assessment

In this action research, assessment was provided as an opportunity to correct and motivate students to produce their best work. Because of the qualitative approach performance assessments are not standardized and the procedure is set according to students' needs.

There is a constant assessment done during the process of this project as a way to guide student to develop critical thinking skill and this enabled me to see what students had learnt and to help them to analyze and link the problem with something visual through these images. Their being able to solve the problem given meant that I could verify their understanding which confirmed that the strategies followed had given them the knowledge to solve word problems.

### Data analysis

Eisner (1991) considered that “information becomes data only if a researcher is able to make it meaningful” (p. 185) therefore, by extracting useful information from the data, that provide important evidence, is the way to create meaning to the essential component of qualitative research. In this order, to answer the research question formulated in this study, I examined the information by reading the collected data several times and by reflecting on them to identify repetitive themes. Thus, I sorted the data according to the research questions obtained by the information that was gathered from all the sources provided by the students, and then I looked for repetitive patterns and meaningful words, through the data that was collected from the surveys, questionnaires, students’ artifacts, and interviews.

Taking into account those events that kept repeating by themselves, like phrases, words used to describe feelings, or responses that seem to emerge, all of this provided great information to be interpreted. Then triangulation was used as Flick (2002) considered it to present “not a tool or a strategy of validation, but an alternative to validation” (p. 230).

These categories were the result obtained in the analysis of data collection, and they emerged through the labeling and the conceptual grouping process, which were:

“Changing perception towards math word problems”, “Adapting a variety of strategies to solve word problems”, and “Developing higher thinking process to deal with the solution of problems”. Once I found the categories and the connection with the theory, I arranged them in the chart below.

Research question	Categories
How does the implementation of visual representation and the recognition of word phrases and key words in mathematics word problems, in an L2 contribute to the development of critical thinking skills?	Changing perception toward word problems improve their understanding on their performance
	Using the Strategies planned
	Developing higher thinking process deal with the solution of word problems

Category 1: Changing perceptions towards math word problems improve their understanding on their performance

Based on the data analysis, it appears that before any kind of intervention students attending academic learning support found word problems difficult; this may be because they did not like them or they had a hard time understanding it. Many stated that they were unsure of how to solve mathematics problem. This perhaps is because they were not clear on how to approach mathematics word problems or what steps to follow, as it showed on students' written reflection (See Appendices F), after working with a problem, here is a clear evidence when all of the students stated the dislike about word problems and it showed clearly the feeling toward word problems. Also it is confirmed the same perception when checking students' answers from the first survey before any intervention had been made. Students attending the mathematics learning support were neglected on working with mathematics word problems as it seem in the following example when they were asked in

the survey about: *“Do you find mathematics word problems easy to solve?”* From the ten students being asked all of them answered “No”.

This reaction was powerful; once again it reflected students’ feelings and their difficulty of word problems. The same in the interview students said that they do not have problems with subtractions or addition, but when it comes to word problems it is not easy and they find it very complicated. Some students even emphasizes that they do not like word problems, and think that they are sent to the learning support department, as a way to improve on solving problems. At the beginning of the intervention I observed that students were unmotivated and when presenting a simple word problem as the following one:

*“Our garden had 16 carrots yesterday. We ate 7 of them for dinner. Today there are 4 more carrots. How many carrots are there now?”*

Out of ten students only three showed interest, the rest of them simply did not care about the result, some decided to add all the numbers as a way to give any kind of answer as a: *27 carrots*. Other just didn’t read carefully, or made a plan any kind of strategy to solve the problem; they only did an operation, with out thinking carefully about the question being asked. They did not recognized key words or important facts, this was evidenced in my observations after students finished with the task as it is viewed in students artifacts, besides that I noticed students do not read for understanding, they just look for the number, put together the numbers and do some kind of operation in order to give a “solution”, but really they did not focused on the problem. Based on the data analysis I could observe as Leader and Middleton (2004) assertion regarding the breach that exist between the perception of the students towards word problems, After some sessions



practicing word problems and selecting carefully important words, I observed students changes toward math problems, especially I noticed their willingness to work on word problems. (See Appendices I ) I noticed that most of them, after really understanding how to deal with word problems, started to change their perception and become more interesting in producing qualify work as it showed in the students artifacts, where they demonstrated a neater work , a plan carried out and a enthusiasm to work at their best. (See Appendix J).

As it is demonstrated in students' artifacts as well as students interviews, where seven students out of ten answered to the question: *Do you find word problems easier now after and why?* This question was formulated after many interventions and this is some of their answers.

Students answered:

Student #1- *"Yes, because I understand a little bit more"*

Student # 5 - *"yes because I understand more the problems" "before the problems were bad a now they a good"*

Student #8- *"easier now because I learning in learning support, the key word and do a movie"*

In the last surveys when they were asked: *Have the feeling about problems solving changed?* The majority nine students out of ten answered "yes". Therefore, this confirmed that students have change their perception about word problems and by changing their opinion it also helped student change their feelings, and be more secure to persevere with the process of solving math problems, as it mentioned by Ethington (1992) that success and positive perceptions about mathematical study are likely to encourage students to approach the subject in a more positive way. As teacher we need to be aware of this important aspect,

as how students are perceive mathematics word problems, and implement alternatives for students to see word problems as an easy and friendly task, rather than something difficult to deal with. Consequently, it was clear having all the evidences, that students felt very insecure about word problems and that the intervention helped a great deal, this is clearly observed in the students' attitudes and in the comment that they made as this one:

Student quote #6 *"Now, I like doing solve a problem and I know what to do"*.

#### Category 2: Using the Strategies Planned

Another category of this data analysis is to see how students adapted the wide variety of strategies such as: drawing pictures, looking for key words, finding important words, crossing out non-important, using logical reasoning in the solution of word problems.

Problem solving requires a procedure of strategies as well as the ability to determine when each strategy would be best to used. Therefore in the data analysis I found that students used the strategies as it is analyzed in the students artifacts, where most of them now have a routine to work out word problems such as to look for key-words and underlining it, as it is observed in the students' artifacts at the middle of the process as well as the end of this intervention, where students were more careful with their presentation of their work, showing evidences of how they work out the problems and as it could be observed carrying out a strategy to solve the problem. (See Appendice K).

In their interviews also they all agreed on the importance of making a plan in order to work out a problem, they mentioned the procedure to solve a problem and made explicit comment on how the drawing had helped them to understand the problem. Even thou, students now carried out a plan, there were some students that still need more

reinforcement in identifying important words or crossing out non- important words from a problem, as it is evidenced in some of the students artifacts where some students either cross out important words or found all the words important, and still are not clear about selected essential information.

From the interviews I can analyzed that most of students are now doing a plan before coming solving a problem, they said that now the make a “movie” that is making the drawing, underlining the key words with a color and looking for important information, they are more conscious about these strategies, which they did not had it before. Here are some of their important statements they made when they were asked about the strategies, “*What are your strategies to solve mathematics word problems?*”

Student #8- “*my strategies...first read the question what I have to do, and after do the picture and do the rest*”.

Student#9- “*yes, because I already know how to do it, I do a reading because it help in doing the problem*”.

Student # 3- “*yes, underlining the key words, and drawing like a movie*”

Here as they mentioned in the interviews and shown in the students artifacts, (See Appendice L) where students made drawing as part of their strategy to solve the word problem, I can see that visual representation was a big input to make word problems more clear for them, all of them mentioned this in their survey and interviews and used drawings in all of their works as part of the process. Drawing was something very important for the students and a good strategy to help them understand word problem, I think that the secret behind this drawing is that it gives student’s a more concrete understanding as it shows in the students artifacts. Finally constructing a “movie” as they mentioned, helps them to develop their critical thinking process in the mathematics. And also the guidance of the

teacher, the students were able to be more aware of the procedures and the importance to carry out strategies on their word- problems. This was appreciated in the students' answers in the surveys questions and in their interview answers:

*Key word helped you to solve the problems?* Eight out of ten agree that it did help them

*I understand word phrases in the word problem.* Five out of ten answered yes.

*Drawing and visualizing the problem help understand the problem?* Nine out of ten answered yes.

In the interview it is clear when I asked them to think carefully upon this question:

*These new strategies helped you? Give a reason?*

*"Yes, the strategies help me, because of the movie, I have a problem and make a movie of that problem, I can understand better than before, because everything is more clear." (student #6).*

*"Yes, the strategies of solving problems are helping me to solve problems, like making the movie..." (student #8)*

As it is shown the students answered "Yes" in all the sources and gave a valid reasons for their answer, students' artifacts shows clearly the strategies carried out as a way to solve the problem, therefore it's clear that these strategies given helped them to develop their critical thinking and that they were able to adapt the different strategies taught as a way to understand, the "movie" as students refers to was the most useful strategy for them, therefore this kind of visualization gave them understanding in the language of the mathematics as it was defined by Keene and Zimmermann(1997) that strategies enable students to make the words on a page to be real and concrete.

Category 3: Develop higher thinking process to deal with solving problems.

The analysis of the information collected from the interviews and students artifacts showed improvement in the acknowledge of solving word problems, they have gained critical thinking skills based on the practical work and by incorporating strategies to help them to develop their thinking process. Rippen, et al. (2002) reported that when students are provided with experience like including strategies that facilitate the assimilation of critical thinking activities, students' critical thinking skills can improve. Student artifact evidenced that they now identify the key word by underlining the key words, and important words, as a way to develop the sequence on how to solve the problem, by listing the information as a way to organize the procedure, then choose the operation for a final result.(See Appendice M) likewise, I noticed that students now do their own self-correcting on their work before handed in their work, here is a where it is noticeable that students applied the strategies and are more self conscious of their own performances. Another evidenced is when they use different alternatives to solve a problem, as using other method as it is evidenced in a word problem that student used a year calendar to reach the answer of the problem. This clearly demonstrated their ability thinking deeply before given an answer.

Second in the interviews student informed, that now they think more on the problem before coming to any solution; this proved that they are developing a higher thinking process.

It seem that students thinking process has being better now when facing word problems as it is shown in their artifacts, the evidence shown on the interview also allowed they are more reflective toward their own learning process, as when they were ask : “ *Are you doing much better when solving word problems?*” ten out of the ten students answered

“yes”, they commented that they are doing much better in the regular classes, some manifested that their grades are much better, some explained their answers giving more supported reasons by saying that “*now they think more about what the problem needs to find out and what to do step to follow*”, as looking for important facts, this proof that students had begun to get involved in critical thinking aspects, to accomplished comprehension, in this aspect Nemat (n.d.) talks about a series of activities that help move students through the path of acquiring critical thinking competences, some of those activities were developed within the sessions, they were the on going action of the students to reflect on their thinking process. Likewise in the survey given after carrying out several activities where the strategies were implemented, and with the guidance of the teacher, the students were able to understand the procedure and put in practice the strategies to solve word problems.

The analysis of the information collected from the interviews showed that they were improvement in acknowledging word problems, mainly in following the strategies carried out in these sessions. Although the students have advanced greatly in their critical thinking abilities toward to understanding and solving mathematics word problems, they are just beginning to get involved on this process. They viewed the process as an interesting way to solve word problems. Therefore, I can conclude that this alternative to develop students’ skill to understand and solve problems it is a significant one, since it makes possible for students to be aware of the words and key words found in a word problem and the visualization of the problem to put it into a real context as a way to improve their word problems comprehension. I have now confirmed through the experience with the sample of students that critical thinking activities help construct the first step for acquiring the skills needed to improve their learning process, as it being recognized by Burble & Berk (1999)

“...For Critical Thinking, it is not enough to know how to seek reasons, truth, and understanding; one must also be impassioned to pursue them rigorously. For Critical Pedagogy, that one can critically reflect and interpret the world is not sufficient; one must also be willing and able to act to change that world. From the standpoint of Critical Pedagogy the Critical Thinking tradition assumes an overly direct connection between reasons and action...” Burble & Berk (1999), like I mentioned although the students have advanced greatly and felt more secure and in their critical thinking abilities, there is a need to continue promoting critical thinking on the students as a way to promote students to become effective at solving problems.

Due to the fact we may not forget that Mathematical reasoning requires analytical thinking, and also requires creative and practical thinking performance. Research has demonstrated that students are capable of powerful mathematical reasoning Grouw (1992). However conventional instruction, rather than building on student's thinking ability about mathematics, centers instead on the memorization of facts, becoming critical thinking crucial to defeat and provide a new way of solving and understanding the language of mathematics. As stated by Van Hiele(1986), creating an understanding of the language in content instruction allows opportunities for learners to acquire a new language through the study of academic disciplines such as mathematics and that also includes activities where students are required to think critically and to be able to build problem solving skills.

Just as it was described in the project, critical thinkers should be taught to think logically, analyze and compare, question and evaluate. In activities that emphasize thinking, the aims must include application, analysis, divergent thinking and opportunities to organize ideas and support value judgments. McMillen (1986).

### Conclusions

Taking into account the research question: How does the implementation of visual representation and the recognition of word phrases and key words in mathematics word problems, in an L2 contribute to the development of critical thinking skills? It is clear that using these strategies on students that attend to learning support department, for a special guide, showed an improvement and acquire a mayor understanding of the problems proposed during their sessions, concerning to mathematics. As Bono (1991) explains thinking can be improved through attention and practice, until the skill is mastered, this means that it is a long term process, that must be constantly carry out, and should be encouraged since middle childhood (7-11 years old). In concordance with Freire's statements (1970 & 1973), producing critical learners is not an easy task, but it can be achieved by engaging the "Pedagogy of Question", another tool to gain critical thinking.

At the same time, it is important to remark that teaching must be meaningful and authentic (Krashen,1982), in other words, students learn mathematics by constructing their own understanding. They learn mathematics by an active participation, and this can be assured by applying strategies and by creating students interest, for this purpose teachers need to know students and students' prior knowledge make them participated in their own learning process. Constructivism is undoubtedly a major theoretical influence in science and mathematics education, due to the fact students need to construct their own understanding of each concept in order to develop their own thinking skill so that the primary role of teaching is not to transfer knowledge, but to create opportunities for students to construct their particular mathematical concept as explained in Piaget's (1951)



theory of knowledge. Therefore, learning focuses on how student's learning is affected by context, beliefs and attitudes. (Vygotsky's , 1978)

It is seen that through the analysis of the students samples and of their personal opinions, their perception towards the mathematics have change in a positive way, finding now word problems easier to target by the learning useful strategies. This could be enlighten because teaching mathematics skills through word problems solving allows students to be focused more on the concept being asked rather than memorizing facts. Due to the fact when working on word problems, the goal for both teachers and students is to select as correctly as possible the operation that will provide the answer (Graeber and Tanenhaus 1993). Through experiences of critical thinking, students select the information required and analyze it in order to come to a conclusion. This is a way to develop their critical thinking techniques to evaluate information and ideas, and as a way to decide what to accept and believe (Kurland, 2000). As a result they obtain a better understanding of the problem and instruction given during the normal classes and in the sessions with learning support department.

On the other hand, the data showed how alternative solutions to problems can be encouraged using logic, reasoning, approximation, estimation and visual representations. It was also clear by the analysis that students use the knowledge that they have about following a strategy before proceeding to answer the problem. Also students demonstrated more understanding when reading a word problem and seem more secure when facing a word problem. Moreover, when student got used to make the drawing to visualize the problem and turned it to a real life situation, in that sense students found the "movie" as they called it, word problems became more motivating and challenging for them to

understand it. As well as looking for key words to guide their procedure. (See appendices G, H, I, J)

Since students from this study have been in contact with the English language, therefore, students have element in order to communicate ideas and understand written word problems. Furthermore, the knowledge had helped to progress in their thinking skill applied in the word problems, It was observed that students convey meaning when they recognize important key words or important words in a word problem, which not only understand the language in the context but construct meaning out of the key word and word phrases, this relationship between the word use for mathematics concept allowed the student to move from the abstract to the concrete with out to much difficulty according to the evidences taken from the from the research.

Based on these results, I can conclude that students attending academic learning support found these activities motivating and that it had help them to deal better in the solution of word problem. If anything the students learned various strategies they can use when faced with a word problem and have gained confidence in themselves. It is also important to recognize teacher role as a facilitator in the process of acquiring the critical thinking skills, as stated by Freire(1970) he suggests that teachers who recognize the possible value of developing critical literacy do not view their students as vessels to be filled, and instead create experiences that offer students opportunities to actively construct knowledge. This is a practice which pushes and challenges the learners to think critically.

Finally, I personally think that these activities were mainly efficient for students attending the mathematics learning support, as well as allowing the development of their skills in analyzing, reasoning, deciding to achieve the aim of this project that is to contribute to the development of their critical thinking skills. Becoming of great importance for the educational institution, which one of its aim is: that students graduated from The English School, have the necessary skills to confront the demands established in a globalized world, where critical thinking becomes an essential part in the puzzle of knowledge.

### Pedagogical Implications

Based on all the previous information, I can say that this project has the following pedagogical implications for students and teacher. As teachers to adopt new strategies in order to enhance students learning by being more like a facilitator in the learning process that help them understand the problems in a more proactive way, where they feel able to face up to new challenges; the environment in classes should allow students to develop their thinking skill, but also needs to allowed students to reflect on their own academic performance. Teacher should be a helper by posing questions to learners and listening to learners' questions. This is a practice which requires to challenges the learners to think critically, and to adopt a critical attitude towards their work. It is extremely vital that teachers have positive beliefs and attitudes towards this process.

This project has given me indication of how important it is to have a conceptual understanding of content in all subjects, especially mathematics. It must be noted that a priority should be to explore the link between students' perceptions and practical work, which provides significant insight for the students gain confidence in their capacities.

As for students they need to pay more attention to their own academics performance, they should be in contact with reading task to increase their intellectual understanding, and take a more active responsibility towards their own learning, by taking risk and willingness to make mistakes and learn from it.

### Further Research

It is interesting to continue a study where student can development their critical thinking skills in all other areas and all subjects, not only in mathematics. Also it would be very interesting to see more thoroughly the individual perception toward any subject taught in schools, as a way to create a motivating environment for students learning process.

### Limitations of the Study

The major limitation of this study was the reduced numbers of students that participate in the study, due to the fact that as a learning support teacher it is one to one assistance, it would be interesting to see how to have a class group involvement. Also time constraint did not allow for additional research component because of school interferences with additional activities like: academic outing, holidays, sport days, cultural days, and so on...

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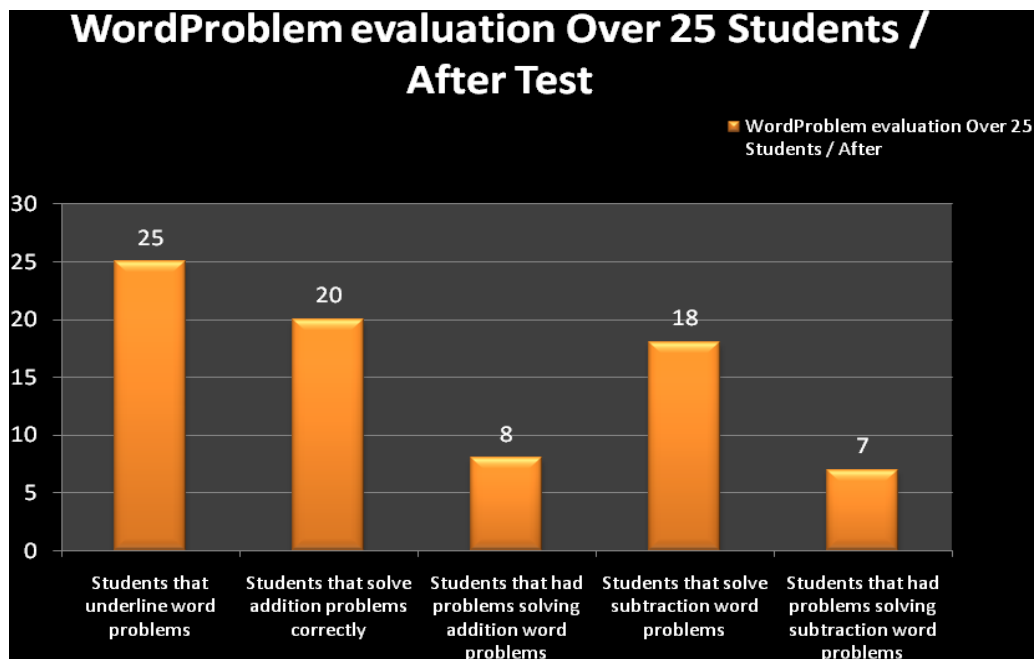
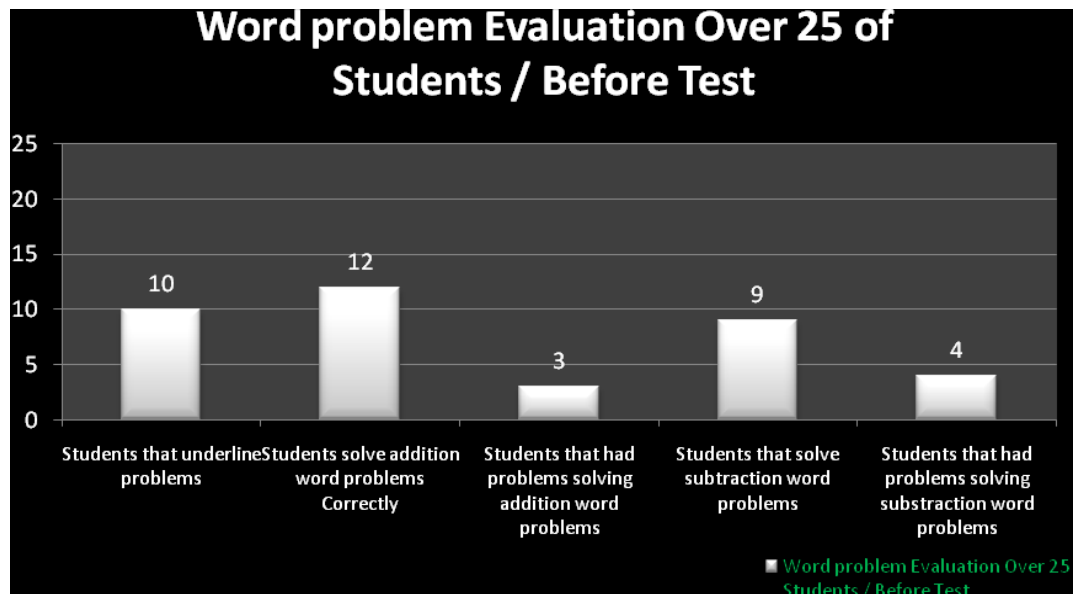
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## Appendix A

Piloting process carried out with 25 student from second grade



**Appendix B**

Bogotá D. C.,

Febrero de 2011

**THE ENGLISH SCHOOL**Señor Luis Eduardo Rivas  
Deputy Headmaster

Apreciados señores:

Por medio de la presente carta solicito me permitan llevar a cabo unos estudios para el trabajo de grado que en este momento estoy realizando en la Universidad San Buenaventura.

La propuesta es incentivar a los estudiantes que asisten al departamento de Learning Support, en el área de las matemáticas a desarrollar habilidades de pensamiento crítico por medio de la solución de problemas, de esta forma se busca potencializar las capacidades de los estudiantes y mejorar su desempeño académico descubriendo sus fortalezas y mejorando sus habilidades. Todo con el fin, de cumplir la misión que tiene la institución educativa formulada en su currículo internacional, donde se busca que los estudiantes se formen de manera pertinente y competitiva frente a las demandas de la sociedad, y construyan una postura crítica y globalizada frente a las problemáticas latentes en la sociedad del conocimiento.

Es importante resaltar que habrá absoluta confidencialidad, y que este proyecto no interrumpirá de ninguna manera el proceso normal de la sesión de clase. Por el contrario será una herramienta para que logren resolver de manera asertiva los ejercicios con las dinámicas propuestas desde el departamento de Learning Support.

Atentamente,

Mabel Patricia Macías Rivas  
Profesora de Learning Support de  
Inglés y Matemáticas en primaria

OW  
P  
Mr. Rivas  
Feb 21-2011

## Appendix C



COLEGIO DE INGLATERRA  
THE ENGLISH SCHOOL

Inscripción 1511  
Lic. Pyme. 02963, 20/11/98  
Rég. Pre-escuela 7222, 04/11/95  
Rég. Prim-Secund 03069, 05/11/97  
Secund Acad. Reg. 02141, 14/11/99  
Secund Acad. - Versión Internacional  
- Reg. 07216, 07/11/99

Bogotá D. C., Febrero de 2011


Estimados Padres de Familia

Reciban un cordial Saludo. Con el fin de llevar a cabo un proyecto sobre el desarrollo del pensamiento crítico de las matemáticas, queremos que su hijo/hija, que actualmente asiste al apoyo para el aprendizaje en el área de las matemáticas, participe en una investigación sobre la comprensión de los problemas en una forma activa, donde logre entender, planear y resolver diferentes tipos de problemas con el fin de desarrollar su pensamiento crítico y entender el lenguaje de las matemáticas.

Este proyecto tiene como fin desarrollar problemas desde varios puntos de vista donde el alumno llegue a encontrar puntos de contacto con los conocimientos adquiridos previamente, esto lo conducirá a desarrollar una aptitud crítica en la resolución de problemas.

La institución, a cargo del Deputy Headmaster tiene pleno conocimiento del trabajo que como docente de learning support estoy realizando y ha aprobado este proyecto de investigación.

Por favor firmar el desprendible y devolverlo a la profesora Mabel Macías.

  
LUIS EDUARDO RIVAS G.  
Deputy Headmaster

  
MABEL MACÍAS  
Profesora de Learning Support

AUTORIZACION

Yo William Delgado  
Con: C.C. 8738.871  
Padres del niño /niña Javier Delgado A.  
Curso: \_\_\_\_\_ Edad: \_\_\_\_\_  
Autorizo para que mi hijo/ hija SI X NO \_\_\_\_\_ participe en el  
proyecto "CRITICAL THINKING THROUGH MATHEMATICS WORD  
PROBLEMS"

Cll. 170 No. 15 - 68 Bogotá, Colombia  
PBX.(571) 6767700 FAX:(571) 6711318  
www.englishschool.edu.co

**Appendix D****SURVEY**

1. Do you like mathematics?  
(a) Yes            (b) a little bit            (c) No
2. Do you find math problems difficult to solve?  
(a) Yes (B) No
3. Do you understand most of the vocabulary in a word problem?  
(a) Yes (B)No
4. Do you need someone to help you in the solution of word problems?  
(a) Yes            (b) most of the time    (c) sometime            (d) No
5. The best thing about learning math is  
\_\_\_\_\_  
\_\_\_\_\_
6. The hardest thing about learning math is  
\_\_\_\_\_  
\_\_\_\_\_
7. When you solve math word problems you usually  
(a) Read the whole word problem  
(b) Try to look for key words  
(c) Give answers but really don't know what to do  
(d) Draw a picture of what they mean in the word problem
8. Do you understand when you read instruction in English  
(A) Most of the time    (b) sometimes    (c) no really
9. Do you need help at home to do your math homework specially when there is a word problem to solve?  
(a) Yes    (b) most of the time    (c) sometimes

## Appendix E

### MATH RUBRIC Students Self- Reflections

From 1 to 4 indicating that: *4-always 3-sometimes 2- never 1- needs to work on it.*

Do you follow a plan before Solving the problem		
Visual word problems in a picture, to help you solve-word problems		
Follow the steps like, underlining “key words” and searching for important, help you solve word problems in a better way		
Do you show all the work		
Do you check the answer		
Do you feel better dealing with word problems		
Do you pay more attention to the “Words” in a word problem		
Do you connect word problems to real life		

## Appendices F

Second grade 2-5

I don't like problems  
because I don't understand

I like multiply and  
divide.

And all the things that  
have numbers

## Appendices G

Name: tulinn david SILVA  
Date: Feb-28-11  
Class: 2-1

SANTY IS DOING A PUZZLE. HE HAS PUT TOGETHER 643 PIECES. THERE ARE 1,000 PIECES IN THE PUZZLE. HOW MANY PIECES ARE LEFT TO PUT TOGETHER?



643  
Pieces



~~1,000~~  
~~643~~  
-----  
0357






## Appendices H

Name: Mateo Escobar CervantesKidZone.4  
Word Problem

The Easter aisle at the local supermarket has been very busy the last two weeks. The supermarket sold 342 chocolate Easter bunnies last week. So far this week they have sold 985 more bunnies.

<p>342 chocolate Easter bunnies last week</p> <p>985 more bunnies</p> <p>342 bunnies</p> <p>985 bunnies</p> 	$\begin{array}{r} 342 \\ + 985 \\ \hline 1327 \end{array}$	<p>they sold 1327 chocolate bunnies</p>
--	--	---

How many chocolate bunnies have been sold in all? 1327

- 1) First I read the sentences and after I underline the important words. 2) I do the movie. 3) now that is clear I can do the operation and give a complete answer

## Appendices I

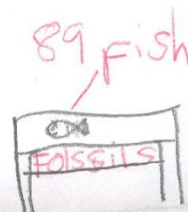
Name: Diego Zubizarreta  
 Date: Feb 22/11  
 Class:

DANNY MADE A DISPLAY OF PLANT AND FISH FOSSILS FOR THE LIBRARY. HE PUT ON 478 PLANT FOSSILS. HE PUT 89 FISH FOSSILS. HOW MANY FOSSILS WERE IN THE DISPLAY?

1<sup>st</sup> step: read it.

2<sup>nd</sup> step: Understand the sentences.

3<sup>rd</sup> step: Make the move.



How many Fossils

$$\begin{array}{r} 478 \\ + 89 \\ \hline 567 \end{array}$$

fossils

4<sup>th</sup> step: read the question

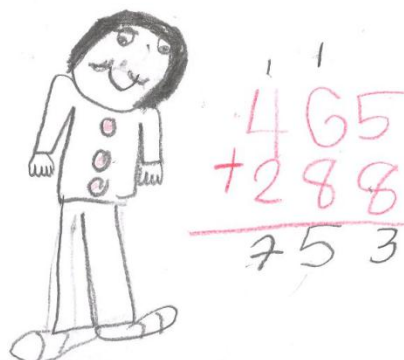
5<sup>th</sup> step: Make the operation

6<sup>th</sup> step: Give a complet answer

## Appendices J

Name: Daniela Capitani  
Date: February 25/2011  
Class: 2-5

MANUEL HAS A COLLECTION OF MINIATURE CARS. THEN HE GAVE 465 CARS TO HIS BROTHER. NOW HE HAS 288 CARS. HOW MANY CARS DID HE HAVE TO BEGIN WITH?



Manuel have to Begin With is 753 Cars.